$$z_{22} = \frac{2.94736\lambda^2 + 0.66321}{\lambda^3 + 3.08303\lambda}.$$

To synthesize the network we will form $z_{22}''(S) = z_{22}/C$ as described in [2] and synthesize the network in the form shown in Fig. 2.

$$z_{22}'' = \frac{2.94736\dfrac{S^2}{S^2+1} + 0.66321}{S\left(\dfrac{S^2}{S^2+1} + 3.08303\right)}$$

$$= \frac{3.61057S^2 + 0.66321}{4.08303S^3 + 3.08303S}$$

$$3.61057S^2 + 0.66321 \;\big)\; 4.08303S^3 + 3.08303S \;\big(\; 1.13085S$$

$$\underline{4.08303S^3 + 0.74994S}$$

$$2.33309S \;\big)\; 3.61057S^2 + 0.66321 \;\big(\; 1.54755S$$

$$\underline{3.61057S^2}$$

$$0.66321 \;\big)\; 2.33309S \;\big(\; 3.51788S.$$

The resultant network is that of Fig. 3 with $C_1 = 3.51788$, $L_2 = 1.54755$, and $C_2 = 1.13085$. L_1 has not been determined but is not necessary. We divide the elements as shown in Fig. 4 to be compatible with Fig. 2 and find that L_1 must equal 0.49747. The resulting transmission-line network is then the network of Fig. 5.

The transmission coefficient of a low-pass or half-wave filter made up of cascaded unit elements is also given by (1) where $x = \sinh \bar{\theta}/jp$ and $p = \sin \theta_c$. $\bar{\theta}$ is a complex angle which on the j axis equals $j\theta$ and x then equals $\sin \theta / \sin \theta_c$. θ_c is the cutoff angle of the low-pass filter and $2\theta_c/\pi$ is the percentage bandwidth of the half-wave filter. For the same value of k and $\sin \theta_c = \cos \theta_0$ ($\theta_c = \pi 2 - \theta_0$) the transmission coefficients are identical except for a translation of 90°. Cristal has shown [7] that the input impedance of one can be derived from the other under this condition by replacing λ by $1/\lambda$. If one has found the denominator polynomial of (11)

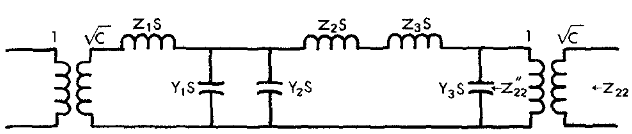

Fig. 2.

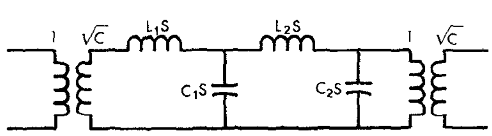

Fig. 3.

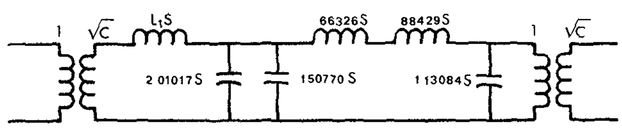

Fig. 4.

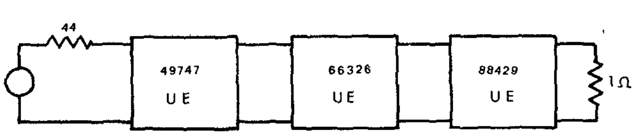

Fig. 5.

$$D_n\lambda^n + D_{n-1}\lambda^{n-1} \cdots + D_0$$

then replacing λ by $1/\lambda$ one obtains the denominator polynomial for the low-pass filter $D_0\lambda^n + D_1\lambda^{n-1} \cdots + D_n$. One then forms z_{22} as the even over the odd part of the polynomial, and synthesizes the network in the S plane exactly as was done in the previous example. Hence if one wants to synthesize a three-section Chebyshev low-pass filter with $k^2 = 1.5777 \times 10^{-4}$ and $\sin \theta_c = 0.464$ then one obtains directly from the coefficients of the denominator polynomial of the previous example that

$$z_{22} = \frac{3.08303\lambda^2 + 1}{0.66321\lambda^3 + 2.94736\lambda}.$$

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Single Hybrid Tee Frequency Discriminator

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Abstract—A novel single hybrid tee frequency discriminator is investigated. It consists of ordinary microwave components and its tuning is achieved by means of a movable short circuit. The discriminator properties are comparable to those of an ordinary phase dis-

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criminator. Because of its simplicity, the discriminator can be used for instantaneous frequency measurements in nanosecond time intervals.

Measurement of instantaneous amplitude and frequency in nanosecond time intervals are needed not only in military applications but also in microwave communications since present semiconductor devices allow modulation rates approaching a gigabit range. Designed for military applications, instantaneous frequency-measuring systems are usually complex and expensive. A novel frequency discriminator, which can be used for instantaneous measurements, is briefly investigated in this letter. It is the simplest version of a phase discriminator where only a single hybrid tee is employed. If proper care is paid to detector circuits, the discriminator can be used for measurements in nanosecond time intervals.

The present instantaneous frequency discriminators [1] are more or less complex versions of phase discriminators [2]–[5]. In all microwave phase discriminators [2], the measured signal is divided by a power splitter (either a 3-dB hybrid or hybrid tee) into two channels commonly consisting of two transmission lines of different electrical length βL_1 and βL_2 (β represents the frequency-dependent phase constant of the lines). The signals from the outputs of the two channels are then combined in a power combiner (usually similar to a power splitter). Because of the signal interference, the envelope of the output signal is proportional to the amplitude and frequency of the measured signal. Square-law detection, unity VSWR (assumed for calculation convenience) results in a net output voltage [2]

$$E_{\text{out}} = E_{d1} \pm E_{d2} = \gamma_d \frac{E_{\text{in}}^2(t)}{2} \cos \beta (L_1 - L_2) \tag{1}$$

where $E_{\text{in}}(t)$ is the voltage amplitude of the measured signal and γ_d is the diode voltage sensitivity (matched diodes are assumed).

The single hybrid tee discrimator is schematically shown in Fig. 1. It consists of an ordinary nonmatched waveguide tee, a transmission line terminated by a movable short circuit, and two crystal detectors which match terminate the E and H arms of the tee. The principle of operation is also simple. The measured signal, represented by an incident normalized wave a_1, enters the hybrid tee via the 1 arm and is split into the H, E, and 2 arms giving rise to emerging waves b_{13}, b_{14}, b_{12}, and b_{11} (the first subscript denotes the number of the wave while the second subscript denotes the arm from which the wave emerges). The wave b_{12} enters the auxiliary transmission line and travels the distance L before being reflected by the line short circuit. The incident wave returning to the hybrid tee via the 2 arm is given by

$$a_2' = -b_{12} \exp - j\beta 2L \tag{2}$$

where β is the frequency-dependent phase constant of the transmission line. The a_2' wave undergoes similar splitting as the wave a_1 gives rise to waves b_{23}, b_{24}, b_{21}, and b_{22}. Since the phase difference of the waves b_{13} and b_{23} as well as b_{14} and b_{24} depends on the factor $(\beta 2L)$, the waves in the E and H arms interfere. The envelope of the signals emerging from the E and H arms, which is also affected by additional reflections between the input of the 2 arm and the short circuit, thus depends on the amplitude and frequency of the measured signal.

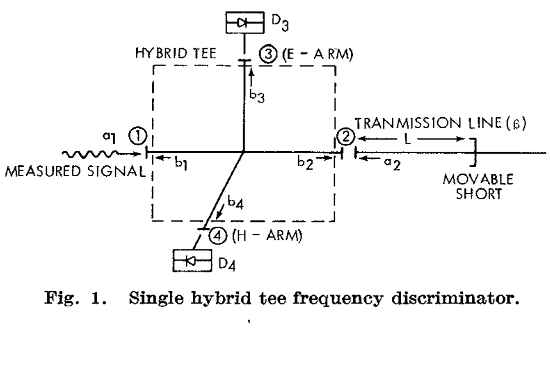

Fig. 1. Single hybrid tee frequency discriminator.

A scattering matrix representation is used for quantitative description of discriminator properties. Because of the isotropy and symmetry conditions, the scattering parameters of the hybrid tee satisfy [6]

$$S_{ij} = S_{ji}(i,j = 1,2,3,4);\; S_{14} = S_{24};\; S_{13} = -S_{23}. \tag{3}$$

Assuming an incident normalized wave a_1, match-terminated E and H arms, and the termination of the 2 arm resulting in the wave a_2, waves emerging from the 1, 3, and 4 arms are

$$b_1 = a_1[S_{11} - DS_{12}^2 \exp - j\beta 2L] \tag{4}$$

$$b_3 = a_1 S_{13}[1 + DS_{12} \exp - j\beta 2L] \tag{5}$$

$$b_4 = a_1 S_{14}[1 - DS_{12} \exp - j\beta 2L] \tag{6}$$

where $D = [1 + S_{22} \exp - j\beta 2L]^{-1} = |D| \exp j\phi \approx 1$. Assuming square-law detection, the discriminator net output voltage is

$$E_{\text{out}} = E_{d3} - E_{d4} = \frac{E_{\text{in}}^2(t)}{2} < (\gamma_{d3}|S_{13}|^2 + \gamma_{d4}|S_{14}|^2)|D||S_{12}|2\cos(\beta 2L - \phi_{12} - \phi) + (\gamma_{d3}|S_{13}|^2 - \gamma_{d4}|S_{14}|^2)(1 + |D|^2|S_{12}|^2) > \tag{7}$$

where γ_{d3} and γ_{d4} are voltage sensitivities of the diodes D_3 and D_4 and ϕ_{12} is the phase angle of the S_{12} parameter. The input VSWR of the discriminator, which can be evalued from (4), varies in the limits

$$\frac{1 - |S_{11}| + |D||S_{12}|^2}{1 + |S_{11}| - |D||S_{12}|^2} < \text{VSWR} < \frac{1 + |S_{11}| + |D||S_{12}|^2}{1 - |S_{11}| - |D||S_{12}|^2} \tag{8}$$

depending on the signal frequency and the length of the transmission line.

The discriminator output voltage depends on the S_{12} parameter according to (7). Since the amplitude of the S_{12} parameter of a lossless waveguide E–H tee is related to the reflection coefficients of the 1, 3, and 4 arms by [7]

$$|S_{12}|^2 = \frac{|S_{33}|^2 + |S_{44}|^2}{2} - |S_{11}|^2 \tag{9}$$

only nonmatched hybrids, usually characterized by $|S_{33}| > 0.2$ and $|S_{44}| > 0.2$, will provide a useful sensitivity. Scattering parameters of a typical hybrid tee depend very little on frequency in a subgigahertz bandwidth (e.g., in the range of 6.3–6.8 GHz, the parameters of a C622A Microlab hybrid tee are as follows: $|S_{11}| \approx |S_{22}| = 0.08 - 0.1$; $|S_{13}| = 0.65 - 0.60$; $|S_{14}| = 0.60 - 0.54$, and $|S_{12}| = 0.46 - 0.52$) and the shape of the discriminator response is determined by the cosine term. Thus the response curve of the single tee discriminator is very similar to that of an ordinary phase discriminator. The voltage nonsymmetry of the response curve, which depends on the S_{13}, S_{14}, and S_{12} parameters, can be compensated by a proper choice of the detector diodes. For the above typical parameters, equal incident power and equal length of the transmission line, the voltage peaks of the single tee discriminator are about 65 percent of the voltage peaks of the ordinary phase discriminator with unity VSWR while the single tee discriminator sensitivity (sensititivity is defined as the slope of the linear part of the response curve) is about 30 percent higher than that of the ordinary discriminator. The input VSWR of the single tee discriminator will be within the limits $1.35 < \text{VSWR} < 2.08$ in the frequency range of 6.3–6.8 GHz. A 6-dB attenuator in the discriminator input line will reduce the input VSWR to $1.08 < \text{VSWR} < 1.20$.

The results of experimental investigations on a single hybrid tee discriminator are graphically shown in Figs. 2–4. To eliminate the effects of signal-dependent sensitivities of detector diodes on the measured response curve, power meters were employed for the measurement of the output signals of the E and H arms. The polar display of the E and H arm output powers is shown in Fig. 2 for various frequencies. The x and y coordinates represent output power normalized to the input power. Crosses and dots indicate experimental

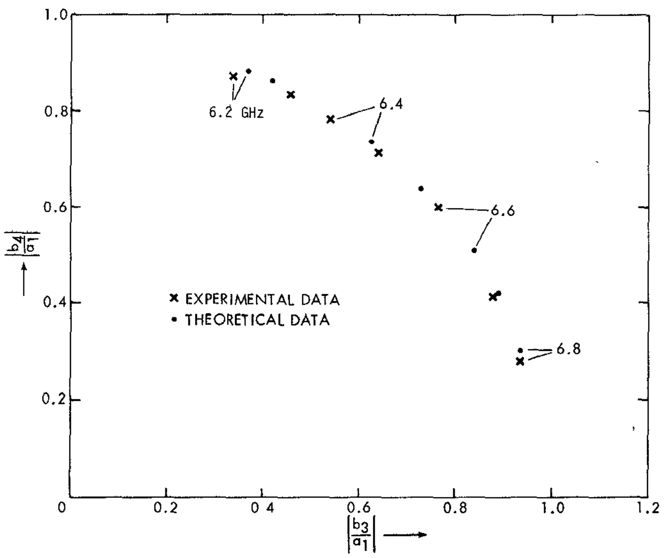

Fig. 2. Polar display of the discriminator response ($P_{in} \approx 10$ mW; $L = 1.78$ cm).

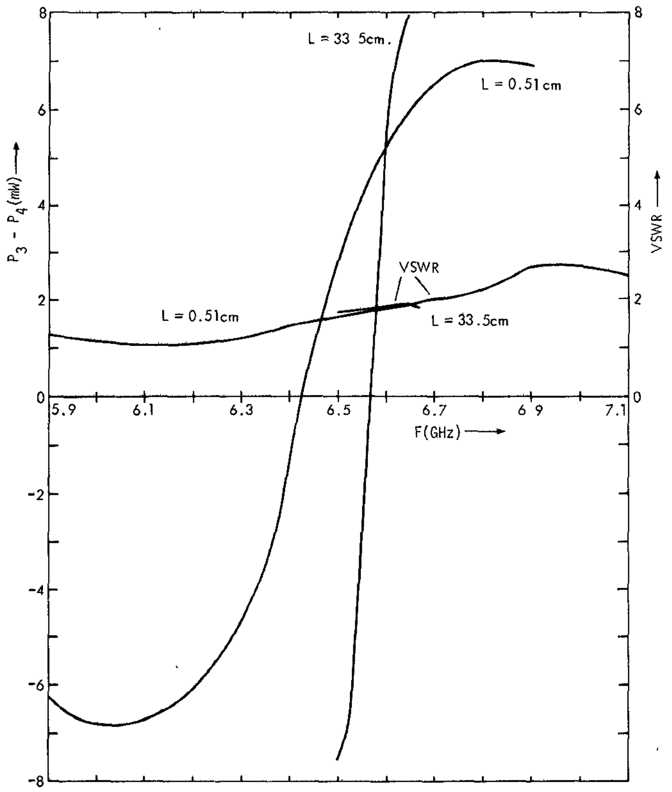

Fig. 3. Experimental response of the single hybrid tee frequency discriminator ($P_{in} = 10$ mW; $L = 0.51$ cm and 33.5 cm).

and theoretical data, respectively. As can be seen, the agreement with respect to absolute values is very good while the phases do not agree that well. This is because of relatively poor accuracy in the measurement of the phase angles of the tee-scattering parameters. In the polar display of the discriminator outputs, the distance and phase angle of a displayed point uniquely determine the instantaneous power and frequency of the measured signal. The discriminator response and input VSWR are shown in Fig. 3 for two different lengths of the transmission line. Power meters instead of detector diodes were used to measure the response curve. Note the slight shift of the curve toward positive voltages. The input VSWR lies within the predicted range. The measured dependence of the discriminator fractional bandwidth on the transmission line length is shown in Fig.

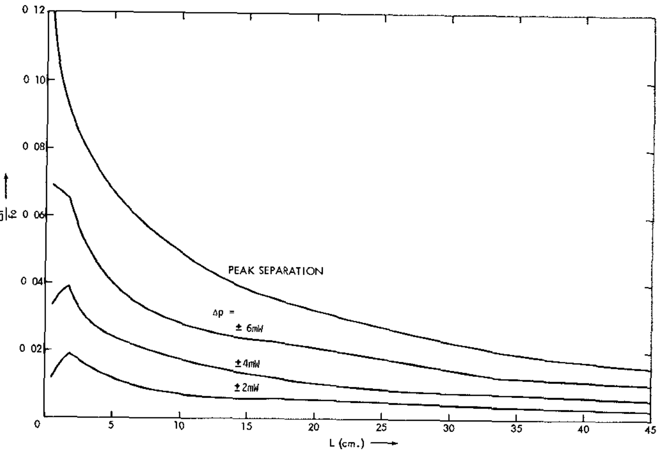

Fig. 4. Experimental fractional bandwidth of the single hybrid tee discriminator ($P_{in} = 10$ mW; $f_0 \approx 6.6$ GHz).

4 (the fractional bandwidth is defined as the ratio of the difference of frequencies at which the discriminator output reaches a given absolute value to the center frequency). Three curves for the ±6-mW, ±4-mW, and ±2-mW bandwidth are displayed. Since the discriminator sensitivity is proportional to the transmission line length, the discriminator bandwidth can be increased only at the expense of the sensitivity. Because of the finite length of the hybrid tee arms, the discriminator fractional bandwidth is limited from above. The bandwidth irregularities at small L are expected to be caused by the interaction of the short circuit with the higher order modes generated on the internal discontinuity of the hybrid tee. According to Fig. 4, the maximum fractional peak separation of about 12 percent can be achieved with the single hybrid tee discriminator.

Let us summarize and compare the performance of the single hybrid tee discriminator with that of the ordinary phase discriminators. The single tee discriminator is very much simpler requiring a bare minimum of ordinary microwave components. It can be quickly assembled for any desired center frequency and any fine tuning or readjustments are extremely simple. Its electrical properties, i.e., sensitivity, peak detected outputs, bandwidth, compare very favorably with those of ordinary phase discriminators. If the hybrids of the power splitters and combiners of ordinary discriminators are not well matched, which is usually the case, even the input VSWR of both discriminators are very much the same. The fact that for a given bandwidth only one-half of the physical length of the ordinary discriminator is required may sometime become a welcome advantage of the single tee discriminator. Since usually $|S_{22}| < 0.1$, the multiple reflections in the short-circuited arm have very little effect on the discriminator response curve. Then the propagation time of the b_{24} and b_{23} waves is the delay time introduced by the discriminator circuitry. This delay time is usually in the subnanosecond range and thus the response time of the single tee discriminator is basically that of the detector circuits.

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